Distributional Learning of Context-Free Grammars and Simple Context-Free Tree Grammars

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December 19, 2012
最先端構文解析とその周辺

Outline

Grammatical Inference

- Algorithmic learning of formal languages
  - String languages
  - Tree languages ...
- More theoretical rather than heuristic
- Motivations/Applications:
  - Mathematical model of natural language acquisition
  - Grammar extraction from tagged/untagged corpora
  - Biological sequences

Regular Language Learning and CFL Learning

- Fruitful positive results on the learning of Regular languages
  - Query learning
  - PAC learning under the simple distribution
  - Identification in the limit from positive and negative data
  - Learning interesting subclasses from positive data only
- Nice properties of Regular languages
  - Myhill-Nerode Theorem – Canonical DFA

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  - Identification in the limit from positive and negative data
  - Learning interesting subclasses from positive data only
- Nice properties of Regular languages
  - Myhill-Nerode Theorem – Canonical DFA
- Few positive results on CFL learning before this century
  - No nice mathematical properties
  - No canonical automata/grammars
- *Distributional Learning* for CFLs

Distributional Learning

- Substitutable CFLs are identifiable in the limit from positive data only (Clark and Eyraud 2007)
- Query learning of c-deterministic/congruential CFGs (Shirakawa & Yokomori 1995 / Clark 2010)
- Quite rich subclasses are identifiable in the limit from positive data and MQs (Clark et al. 2009, Clark 2010, Yoshinaka 2010, 2011, 2012 etc.)
- PAC learnability of Unambiguous NTS languages (Clark 2006)

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Heuristics has preceded Theory
(Brill et al. 1990, Adriaans 1999, van Zaanen 2000, Klein & Manning 2002, etc.)

Outline

Notation

A Context-Free Grammar

A tuple $\langle \Sigma, V, I, R \rangle$

- Σ: finite set of terminal symbols
- V: finite set of nonterminal symbols
- $I \subseteq V$: set of initial symbols
- R: set of productions

Bottom-up derivation:

- $\alpha N \gamma \Rightarrow \alpha \beta \gamma$ if $N \rightarrow \beta \in P$
- $\mathcal{L}(G, N) = \{ w \in \Sigma^* \mid N \overset{*}{\Rightarrow} w \}$
- $\mathcal{L}(G) = \bigcup_{S \in I} \mathcal{L}(G, S)$

Context

A context is just a pair of strings $l\Box r$ with $l, r \in \Sigma^*$.

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- $L \oslash u = \{\, l\Box r \mid lur \in L \,\}$ and $L \oslash (l\Box r) = \{\, u \mid lur \in L \,\}$,
- Special context $\Box$:
  - $\Box \odot u = u$,
  - $u \in L \iff \Box \in L \oslash u$.

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Syntactic congruence

$$u \equiv_L v \quad \text{iff} \quad L \oslash u = L \oslash v$$

Let $[u] = \{\, v \in \Sigma^* \mid v \equiv_L u \,\}$.

Example

For $L = \{ a^n b^n \mid n \geq 0 \}$,

	$\Box$	$a\Box$	$\Box b$	$a\Box bb$	$\Box abb$
λ	1	0	0	0	0
a	0	0	1	1	1
b	0	1	0	0	0
ab	1	0	0	0	0
aab	0	0	1	1	0
$aaabb$	0	0	1	1	0

$$L \oslash aab = L \oslash aaabb = \{\Box b, a\Box bb, \dots, a^k\Box b^{k+1}, \dots\}$$

$$[aab] = \{ a^{k+1} b^k \mid k \geq 1 \}.$$

$$\forall l\Box r,\ l\Box r \odot aaabb \in L \text{ iff } l\Box r \odot aab \in L \text{ iff } l\Box r \odot a^{k+1}b^k \in L.$$

Keywords of Distributional Learning

- Observe, model, exploit the relation between "substrings" and "contexts" (Observation table indexed by strings and contexts)
- Objectivity
- Symmetric approaches

Outline

Congruential CFGs [Clark 2010]

Congruential context-free grammars

For every nonterminal N of G, if $u, v \in \mathcal{L}(G, N)$, then $u \equiv_{\mathcal{L}(G)} v$.

- If G is congruential, and we binarize G, then the result is congruential.
- So we assume productions are all like $N \to PQ$ or $N \to a$.

Examples

- $L = \{ a^n b^n \mid n \geq 0 \}$ with $S_1 \to \lambda$, $S_2 \to aS_2b$, $S_2 \to ab$.
  - S_1: $L \oslash \lambda = \{ \Box, a\Box b, ab\Box, \Box ab, \dots \} = \{ u\Box v \mid uv \in L \}$
  - S_2: $L \oslash ab = L \oslash aabb = \{ \Box, a\Box b, aa\Box bb, \dots \} = \{ a^n\Box b^n \mid n \in \mathbb{N} \}$
- Dyck grammar: $S \to SS$, $S \to aSb$, $S \to \lambda$.
- Every regular language is generated by a congruential CFG

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Examples not generated by congruential CFGs

- Palindromes: $\{ w \mid w = w^R \}$.
- $\{ a^n b^n \mid n \geq 0 \} \cup \{ a^n b^{2n} \mid n \geq 0 \}$
- $\{ a^m b^n \mid m \leq n \}$

Clark 2010 (modified)

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Identification in the limit

- Input: Infinite sequence of the elements $w_1, w_2, \ldots$ of the learning target L_* in an arbitrary order
- Each time the learner gets an example w_i, it outputs a grammar G_i as her conjecture. After some point the conjecture should be stable and represent the target.

Membership Queries (MQs)

- Q: $w \in \Sigma^*$?
- A: Yes (if $w \in L_*$) ;
  No (otherwise).

Grammar Construction

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- $K \subseteq \mathrm{Sub}(D)$, where $\mathrm{Sub}(D) = \{\, u \mid \exists l, r.\ lur \in D \,\}$,
- $F \subseteq \mathrm{Con}(D)$, where $\mathrm{Con}(D) = \{\, l\Box r \mid \exists u.\ lur \in D \,\}$.

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$G_{K,F} = (\Sigma, V_K, I_K, R_K \cup R_{K,F})$ where

$$V_K = \{\, [\![u]\!] \mid u \in K \,\}.$$

We want $[\![u]\!]$ to generate all and only v s.t. $v \equiv_{L_*} u$,
i.e., $\mathcal{L}(G_{K,F}, [\![u]\!]) = [u]$.

Congruential context-free grammars

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$G_{K,F} = (\Sigma, V_K, I_K, R_K \cup R_{K,F})$ where

- $V_K = \{\, [\![u]\!] \mid u \in K \,\}$,
- $I_K = \{\, [\![u]\!] \mid u \in L_* \,\}$ (by MQ),
- $R_K = \{\, [\![a]\!] \to a \mid a \in \Sigma \cup \{\lambda\} \,\} \cup \{\, [\![uv]\!] \to [\![u]\!][\![v]\!] \mid uv \in K \,\}$,
- $R_{K,F} = \{\, [\![u]\!] \to [\![v]\!] \mid (L_* \oslash u) \cap F = (L_* \oslash v) \cap F \,\}$,

with the aid of *Membership Queries*.

Monotonicity

Hypothesis grammar: $G_{K,F}$

- $V_K = \{ [\![u]\!] \mid u \in K \}$,
- $I_K = \{ [\![u]\!] \mid u \in L_* \}$ (by MQ),
- $R_K = \{ [\![a]\!] \to a \mid a \in \Sigma \cup \{\lambda\} \} \cup \{ [\![uv]\!] \to [\![u]\!][\![v]\!] \mid uv \in K \}$,
- $R_{K,F} = \{ [\![u]\!] \to [\![v]\!] \mid (L_* \oslash u) \cap F = (L_* \oslash v) \cap F \}$.

Monotonicity

If $K \subseteq K'$ then every rule of $G_{K,F}$ is a rule of $G_{K',F}$,
so $\mathcal{L}(G_{K,F}) \subseteq \mathcal{L}(G_{K',F})$.

Anti-Monotonicity

If $F \subseteq F'$ then every rule of $G_{K,F'}$ is a rule of $G_{K,F}$,
so $\mathcal{L}(G_{K,F}) \supseteq \mathcal{L}(G_{K,F'})$.

Chain Rule

$$R_{K,F} = \{ [\![u]\!] \to [\![v]\!] \mid (L_* \oslash u) \cap F = (L_* \oslash v) \cap F \}$$

	$\Box$	$a\Box$	$\Box b$
λ	1	0	0
a	0	0	1
b	0	1	0
ab	1	0	0
aab	0	0	1
abb	0	1	0

We have
$[\![a]\!] \leftrightarrow [\![aab]\!] \in R_{K,F}$
as they *look* congruent.

Anti-Monotonicity

If $F \subseteq F'$ then $R_{K,F} \supseteq R_{K,F'}$, and thus $\mathcal{L}(G_{K,F}) \supseteq \mathcal{L}(G_{K,F'})$.

Chain Rule

$$R_{K,F} = \{ [\![u]\!] \to [\![v]\!] \mid (L_* \oslash u) \cap F = (L_* \oslash v) \cap F \}$$

	$\square$	$a\square$	$\square b$	$\square abb$
λ	1	0	0	0
a	0	0	1	1
b	0	1	0	0
ab	1	0	0	0
aab	0	0	1	0
abb	0	1	0	0

NO $[\![a]\!] \leftrightarrow [\![aab]\!]$ any more.

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Example

	$\square$	$a\square$	$\square b$	$\square ab$	$\square abb$	$aab\square$
λ	1	0	0	1	0	0
a	0	0	1	0	1	0
b	0	1	0	0	0	1
ab	1	0	0	0	0	0
aab	0	0	1	0	0	0
abb	0	1	0	0	0	0
$aabb$	1	0	0	0	0	0

$$[\![aabb]\!] \in I_K$$

$$[\![aabb]\!] \Rightarrow [\![a]\!][\![abb]\!] \Rightarrow [\![a]\!][\![ab]\!][\![b]\!] \Rightarrow a[\![ab]\!]b \Rightarrow a[\![aabb]\!]b \Rightarrow aaabbb$$

$$aaabb \in \mathcal{L}(G_{K,F}),\ \text{in fact } \mathcal{L}(G_{K,F}) = \{\, a^n b^n \mid n \geq 0 \,\}$$

Soundness

We have $[\![u]\!] \to [\![v]\!]$ if $(L_* \oslash u) \cap F = (L_* \oslash v) \cap F$.

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Every K admits finite F s.t. $G_{K,F}$ has no incorrect rules.

Proof. For each $u, v \in K$, if $L_* \oslash u \neq L_* \oslash v$, then there is $l\Box r \in (L_* \oslash u) \triangle (L_* \oslash v)$. Put $l\Box r$ into F.

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If $G_{K,F}$ has no incorrect rules, $\mathcal{L}(G_{K,F}) \subseteq L_*$.

Proof.

- $[\![a]\!] \to a \quad \cdots \quad a \in [a]$,
- $[\![uv]\!] \to [\![u]\!][\![v]\!] \quad \cdots \quad [uv] \supseteq [u][v]$,
- $[\![u]\!] \to [\![v]\!] \quad \cdots \quad [u] = [v]$ since the rule is correct,

Hence $[\![u]\!] \overset{*}{\Rightarrow} v$ implies $v \in [u]$.

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Hence $[\![u]\!] \overset{*}{\Rightarrow} v$ implies $v \in [u]$.
Particularly for $[\![u]\!] \in I_K$, we have $v \in [u] \subseteq L_*$.

Completeness

Suppose L_* is generated by a congruential CFG G_*.

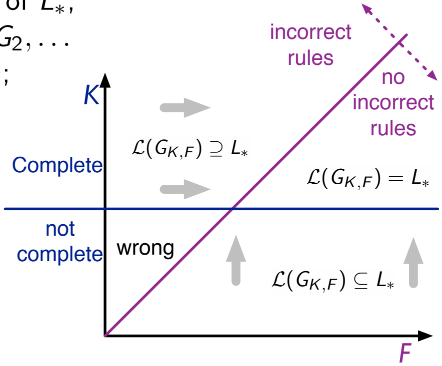
If $K \cap \mathcal{L}(G, \alpha) \neq \emptyset$ for every rule $N \to \alpha$ of G_*, then $L_* \subseteq \mathcal{L}(G_{K,F})$.

Proof. For a rule $N \to PQ$ of G_*, let $v_N, v_P, v_Q \in K$ be the shortest in $\mathcal{L}(G_*, N), \mathcal{L}(G_*, P), \mathcal{L}(G_*, Q)$, resp. Moreover we have $u_P u_Q \in K \cap \mathcal{L}(G_*, PQ)$.
$G_{K,F}$ has $[\![v_N]\!] \Rightarrow [\![v_P]\!][\![v_Q]\!]$ since

$$\begin{aligned}
[\![v_N]\!] &\to [\![u_P u_Q]\!] \text{ by } [v_N] = [u_P u_Q], \\
[\![u_P u_Q]\!] &\to [\![u_P]\!][\![u_Q]\!], \\
[\![u_P]\!] &\to [\![v_P]\!] \text{ by } [u_P] = [v_P], \\
[\![u_Q]\!] &\to [\![v_Q]\!] \text{ by } [u_Q] = [v_Q].
\end{aligned}$$

Learning Algorithm

Data: Positive data $w_1, w_2, \dots$ of L_*;
Result: Sequence of CFGs $G_1, G_2, \dots$
let $K := \emptyset$; $F := \emptyset$; $\hat{G} := G_{K,F}$;
for $n = 1, 2, \dots$ **do**
  let $D := \{w_1, \dots, w_n\}$;
  let $F := \mathrm{Con}(D)$;
  if $D \not\subseteq \mathcal{L}(\hat{G})$ **then**
    let $K := \mathrm{Sub}(D)$;
  end if
  output $\hat{G} = G_{K,F}$ as G_n;
end for



Distributional Learning

- Observe, model, exploit the relation between "substrings" and "contexts"
- [Primal] Learner for congruential CFGs uses Strings for nonterminals and Contexts for removing incorrect rules,
- [Dual] Use contexts for nonterminals and strings for removing incorrect rules.

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- [Dual] Use contexts for nonterminals and strings for removing incorrect rules.
- Further generalization: each nonterminal is represented by sets rather than a single object.

	Primal	Dual
Nonterminal	string / set of strings	context / set of contexts
Rule validation	contexts	strings

Outline

Congruence on Sets

We have defined ...

- $l\Box r \odot u = lur$,
- $L_* \oslash u = \{\, l\Box r \mid (l\Box r) \odot u \subseteq L_* \,\}$,
- $L_* \oslash l\Box r = \{\, u \mid (l\Box r) \odot u \subseteq L_* \,\}$.
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For string set S and context set C, define

- $C \odot S = \{\, lur \mid l\Box r \in C \text{ and } u \in S \,\}$,
- $L_* \oslash S = \{\, l\Box r \mid (l\Box r) \odot S \subseteq L_* \,\}$,
- $L_* \oslash C = \{\, u \mid C \odot u \subseteq L_* \,\}$,
- $S \equiv_{L_*} T$ iff $L_* \oslash S = L_* \oslash T$.

Example

$L_* = \{ a^n b^n \mid n \geq 0 \}$.

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- $L_* \oslash a = \{ \Box b, a\Box bb, \Box abb, \dots, a^i \Box a^j b^{i+j+1}, \dots \}$,
- $L_* \oslash aab = L_* \oslash \{aab, a\} = \{ \Box b, a\Box bb, \dots, a^k \Box b^{k+1}, \dots \}$,
- $\{aab\} \equiv_{L_*} \{a, aab, aaabb\} \not\equiv_{L_*} \{a\}$.

Learning Target

k-Kernel Property (Yoshinaka 2011)

A CFG G has *the k-KP* iff
every nonterminal N admits a finite set $S_N \subseteq \Sigma^*$ such that

- $|S_N| \leq k$,
- $S_N \equiv_{\mathcal{L}(G)} \mathcal{L}(G, N)$.

(Every congruential CFG has the 1-KP but not vice versa.)

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Examples

- Every grammar G with a single nonterminal S has the 1-CP, since the initial symbol S is characterized by $C_S = \{\square\}$: $\mathcal{L}(G) \oslash \square = \mathcal{L}(G) = \mathcal{L}(G, S)$.
  - E.g. $\{ a^n b^n \mid n \geq 0 \}$,
    Palindrome $\{ w \in \Sigma^* \mid w = w^R \}$,
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- $\{a^n b^n \mid n \in \mathbb{N}\} \cup \{a^n b^{2n} \mid n \in \mathbb{N}\}$ has the 2-CP but not 1-CP.
    - $S_1 \to aS_1 b$, $S_1 \to \lambda$, $S_2 \to aS_2 bb$, $S_2 \to \lambda$.
    - $C_{S_1} = \{\square, a\square b\}$ and $C_{S_2} = \{\square, a\square bb\}$.
    - Note $\{a\square b\}$ does not characterize $\mathcal{L}(G, S_1)$ since $abbb \in (L \oslash a\square b) - \mathcal{L}(G, S_1)$.

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  - Note $\{a\square b\}$ does not characterize $\mathcal{L}(G, S_1)$ since $abbb \in (L \oslash a\square b) - \mathcal{L}(G, S_1)$.
- Every regular language has the 1-CP.

Theorem

Theorem (Clark 2010, Yoshinaka 2011)

The class of CFGs with the k-CP is "efficiently" identifiable in the limit from positive data and MQs.

(k is known to the learner)

Grammar Construction

$D \subseteq L_*$: given set of positive examples.

- $F \subseteq \mathrm{Con}(D)$ and $K \subseteq \mathrm{Sub}(D)$.

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If $F \subseteq F'$ then every rule of $G_{K,F}$ is a rule of $G_{K,F'}$,
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Anti-Monotonicity

If $K \subseteq K'$ then every rule of $G_{K',F}$ is a rule of $G_{K,F}$,
so $\mathcal{L}(G_{K,F}) \supseteq \mathcal{L}(G_{K',F})$.

Correctness

We have $[\![C_0]\!] \to [\![C_1]\!][\![C_2]\!]$ if $C_0^{(\Sigma^*)} \supseteq C_1^{(K)} C_2^{(K)}$,
where $C^{(K)} = (L_* \oslash C) \cap K$.

We say that $[\![C_0]\!] \to [\![C_1]\!][\![C_2]\!]$ is *incorrect* iff $C_0^{(\Sigma^*)} \not\supseteq C_1^{(\Sigma^*)} C_2^{(\Sigma^*)}$.

Soundness Lemma

Every F admits finite K s.t. $G_{F,K}$ has no incorrect rules, in which case $\mathcal{L}(G_{F,K}) \subseteq L_*$.

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Soundness Lemma

Every F admits finite K s.t. $G_{F,K}$ has no incorrect rules, in which case $\mathcal{L}(G_{F,K}) \subseteq L_*$.

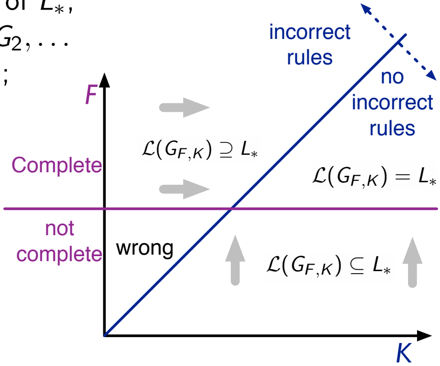
Suppose L_* is generated by a CFG G_* with the k-CP.
That is, every nonterminal N of G_* admits a context set C_N of cardinality at most k s.t. $L_* \oslash C_N \equiv_{L_*} \mathcal{L}(G_*, N)$.

Completeness Lemma

If $C_N \subseteq F$ for every N of G_*, then $L_* \subseteq \mathcal{L}(G_{F,K})$.

Learning Algorithm

Data: Positive data $w_1, w_2, \ldots$ of L_*;
Result: Sequence of CFGs $G_1, G_2, \ldots$
let $F := \varnothing$; $K := \varnothing$; $\hat{G} := G_{F,K}$;
for $n = 1, 2, \ldots$ **do**
  let $D := \{w_1, \ldots, w_n\}$;
  let $K := \mathrm{Sub}(D)$;
  if $D \not\subseteq \mathcal{L}(\hat{G})$ **then**
    let $F := \mathrm{Con}(D)$;
  end if
  output $\hat{G} = G_{F,K}$ as G_n;
end for



Theorems

Theorem (Clark 2010, Yoshinaka 2011)

The class of CFGs with the k-CP is "efficiently" identifiable in the limit from positive data and MQs.

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Combined Property (Yoshinaka 2012)

Every nonterminal N admits either an m-kernel or n-context:

- a finite set $S_N \subseteq \Sigma^*$ s.t. $|S_N| \leq m$ and $S_N \equiv_{\mathcal{L}(G)} \mathcal{L}(G, N)$, or
- a finite set $C_N \subseteq \Sigma^* \square \Sigma^*$ s.t. $|C_N| \leq n$ and $\mathcal{L}(G) \oslash C_N \equiv_{\mathcal{L}(G)} \mathcal{L}(G, N)$

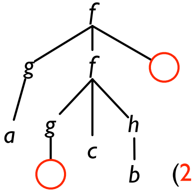
Outline

Simple Context-Free Tree Grammars

- Tree version of context-free (string) grammars
- (essentially) more general than Tree Adjoining (Substitution) Grammars
- (CFG) CF derivation trees yield strings
- (SCFTG) CF derivation trees yield trees

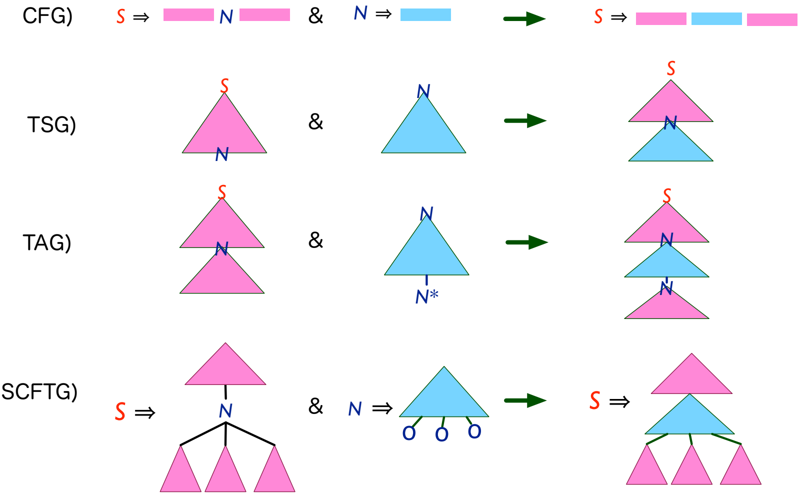
Trees and Stubs

- A ranked alphabet: $\Sigma = \bigcup_{0 \leq i \leq r} \Sigma_i$,
- If $t_1, \ldots, t_k$ are trees and $f \in \Sigma_k$ then $f(t_1, \ldots, t_k)$ is a tree,
- special symbol O of rank 0, which is a “hole”,
- a *k-stub* is a tree t over $\Sigma \cup \{O\}$ that contains exactly k holes, (0-stub = usual tree)
- each nonterminal of a CFG derives strings
- each nonterminal of rank k of an SCFTG derives k-stubs



(2-Stub)

Derivations of Different Formalisms



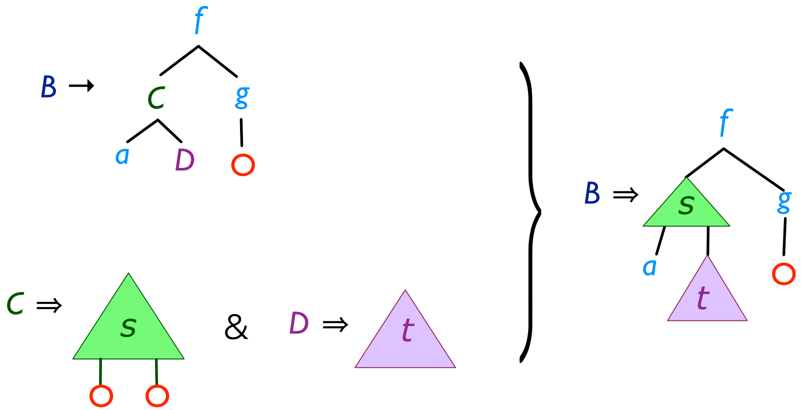
r-Simple Context-free Tree Grammar

A tuple $\langle \Sigma, V, I, R \rangle$ where

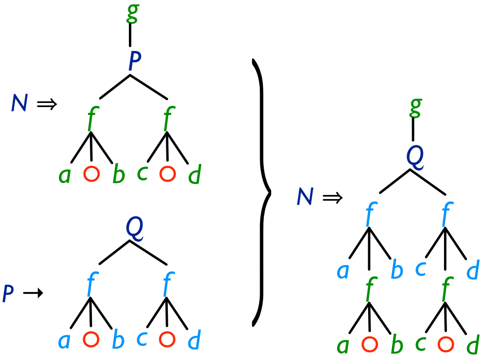
- Σ, V: finite set of ranked terminal/nonterminal symbols,
- $0 \leq \mathrm{rank}(a) \leq r$ for all $a \in \Sigma, V$,
- $I \subseteq V_0$: set of initial symbols (rank 0)
- $R \subseteq \bigcup_{k \leq r} V_k \times (k\text{-stubs})$: set of productions

$\mathcal{L}(G) = \bigcup_{S \in I} \mathcal{L}(G, S)$

Derivation of SCFTG



Example



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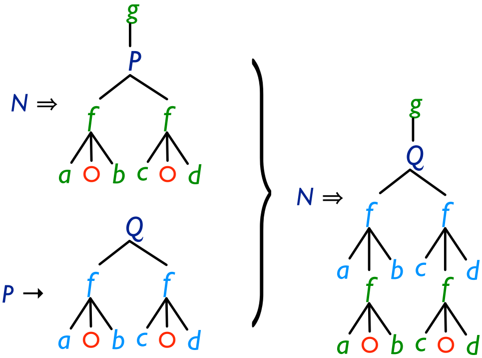

Typical String languages of 2-SCFTGs:
$\{ a^n b^n c^n d^n \mid n \geq 1 \}$,
$\{ a^n b^n c^n d^n e^n f^n \mid n \geq 1 \}$ – not generated by a TAG

Substructure/Context Decomposition

CFGs

- $S \Rightarrow uAv$
- $A \Rightarrow w$
- $uwv \in L(G)$

  $u\square v$: **context**

  w : **substring**

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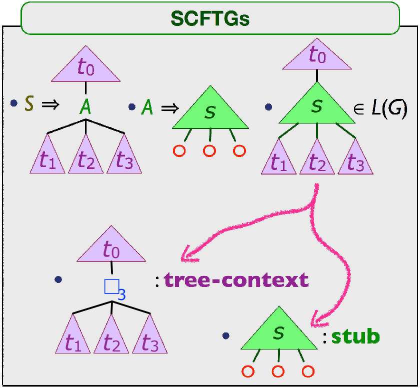

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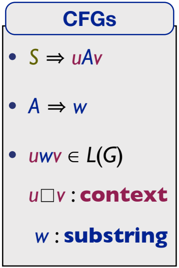

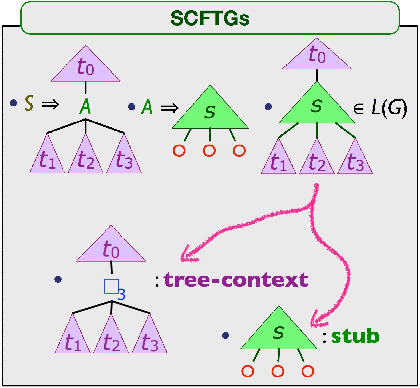

- k-tree context = tree with a hole $\square_k$ of rank k

Every technique on the distributional learning of CFGs can be translated and applied to the learning of SCFTGs!

Congruential SCFTGs

Congruential simple context-free tree grammars

An SCFTG G is said to be *congruential* if it satisfies the following: For any nonterminal N of rank k, if $s, t \in \mathcal{L}(G, N)$, then $s \equiv_{\mathcal{L}(G)} t$, that is, for any k-tree-context c, $c \odot s \in \mathcal{L}(G)$ iff $c \odot t \in \mathcal{L}(G)$.

Grammar Construction

$D \subseteq \Sigma^*$: finite set of tree examples

The learner's hypothesis G is computed from sets K_k, F_k with $0 \leq k \leq r$:

- $K_k \subseteq \mathrm{Sub}_k(D)$, where
  $\mathrm{Sub}_k(D)$ is the set of k-stubs extracted from D,
- $F_k \subseteq \mathrm{Con}_k(D)$, where
  $\mathrm{Con}_k(D)$ is the set of k-tree-contexts extracted from D.

$G = (\Sigma, V, I, R_K \cup R_{K,F})$ where

- $V = \bigcup_{k \leq r} V_k$ with $V_k = \{ [\![s]\!] \mid s \in K_k \}$,
- $I = \{ [\![t]\!] \mid t \in L_* \cap K_0 \}$ (by MQ),
- $R_K = \{ [\![f(\circ, \dots, \circ)]\!] \to f(\circ, \dots, \circ) \mid f \in \Sigma \}$
  $\cup \{ [\![s_0]\!] \to [\![s_1]\!](\circ, \dots, \circ, [\![s_2]\!](\circ, \dots, \circ), \circ, \dots, \circ)$
  $\mid s_0 = s_1(\circ, \dots, \circ, s_2(\circ, \dots, \circ), \circ, \dots, \circ) \}$,
- $R_{K,F} = \{ [\![s]\!] \to [\![t]\!] \mid c \odot s \in \mathcal{L}(G)$ iff $c \odot t \in \mathcal{L}(G)$ for all $c \in F \}$,

Learning Algorithm

Data: Positive data $t_1, t_2, \ldots$ of L_*;
Result: Sequence of SCFTGs $G_1, G_2, \ldots$
let $K := \varnothing$; $F := \varnothing$; $\hat{G} := G_{K,F}$;
for $n = 1, 2, \ldots$ **do**
  let $D := \{t_1, \ldots, t_n\}$;
  let $F_k := \mathrm{Con}_k(D)$
    for $k = 0, \ldots, r$;
  if $D \not\subseteq \mathcal{L}(\hat{G})$ **then**
    let $K_k := \mathrm{Sub}_k(D)$
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  end if
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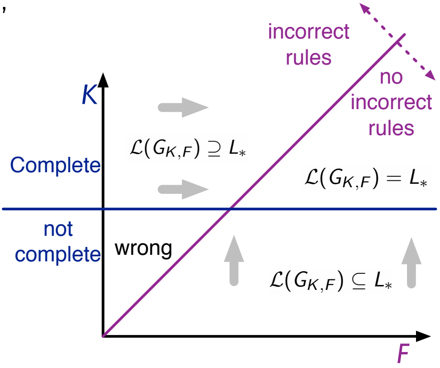

Other Properties and Learning

Similarly one can define k-KP and k-CP for SCFTGs
and design learning algorithms for the corresponding classes.

Other “context-free” formalisms

$$N \rightarrow \alpha[P_1, \dots, P_k]$$

- Context-free grammars (Clark & Eyraud’07 etc....)
  - $N \rightarrow$ string
- Simple context-free tree grammars (Kasprzik & Yoshinaka ’11)
  - $N \rightarrow$ tree/stub
- Multiple CFGs (Yoshinaka ’12 etc.)
  - $N \rightarrow$ tuple of strings
- Linear context-fee linear λ-grammars (Yoshinaka & Kanazawa’11)
  - $N \rightarrow \lambda$-term

Context-free grammar with Montague semantics

$$S(w_1 w_2, Z_1 Z_2) \mathrel{:-} NP(w_1, Z_1) VP(w_2, Z_2),$$
$$VP(w_1 w_2, \lambda x. Z_2(\lambda y. Z_1 y x)) \mathrel{:-} V(w_1, Z_1) NP(w_2, Z_2),$$
$$NP(w_1 w_2, Z_1 Z_2) \mathrel{:-} Det(w_1, X_1) N(w_2, Z_2),$$
$$NP(\text{John}, \lambda u. u\, \textsc{John}) \mathrel{:-},$$
$$V(\text{found}, \lambda yz. \textsc{find}\, yz) \mathrel{:-},$$
$$Det(\text{a}, \lambda uv. \textsc{Intersect}\, uv) \mathrel{:-},$$
$$N(\text{unicorn}, \lambda y. \textsc{unicorn}\, y) \mathrel{:-}$$

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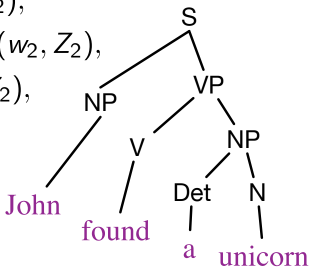

$$\langle \text{John found a unicorn}, \textsc{Intersect}(\lambda y. \textsc{unicorn}\ y)(\lambda y. \textsc{find}\ y\ \textsc{John})\rangle$$

Summary

Distributional Learning

- Context-substring relation
- Primal-dual approaches
- Correctness of rules
- Monotonicity with respect to the two sets

	Primal	Dual
Nonterminal	string / set of strings	context / set of contexts
Rule validation	contexts	strings

Other formalisms

- Simple context-free tree grammars (Kasprzik & Yoshinaka '11)
- Multiple CFGs (Yoshinaka '12 etc.)
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